Année scolaire : 2026 – 2027

CORRECTION DU DEVOIR DE CONTRÔLE N°2

Épreuve : Mathématiques | Classe : 3ème Sciences Techniques | Barème : 20 points

Exercice 1 : Limites de fonctions (5 points)

5 pts

1) a) $D_f = \mathbb{R}$; $\lim\limits_{x\to+\infty} f(x) = \lim\limits_{x\to+\infty} 2x^3 = \boxed{+\infty}$.

b) $D_f = \mathbb{R}$; $\lim\limits_{x\to+\infty} f(x) = \lim\limits_{x\to+\infty} \dfrac{x^2}{x^2} = \boxed{1}$.

c) $D_f = [0\,;+\infty[$; $\lim\limits_{x\to+\infty} x\left(1 - \dfrac{1}{\sqrt{x}}\right) = \boxed{+\infty}$.

d) $D_f = [0\,;+\infty[$; $\lim\limits_{x\to+\infty} \sqrt{\dfrac{x^3}{x^2+1}} = \lim\limits_{x\to+\infty} \sqrt{x} = \boxed{+\infty}$.

e) $D_f = [0\,;+\infty[$; $\lim\limits_{x\to+\infty} (\sqrt{x+1} - \sqrt{x}) = \lim\limits_{x\to+\infty} \dfrac{1}{\sqrt{x+1}+\sqrt{x}} = \boxed{0}$.

2) a) $\lim\limits_{x\to1} \dfrac{(x+2)-3}{(x-1)(\sqrt{x+2}+\sqrt{3})} = \lim\limits_{x\to1} \dfrac{1}{\sqrt{x+2}+\sqrt{3}} = \boxed{\dfrac{1}{2\sqrt{3}}}$.

b) Comme $1 - \cos(2x) = 2\sin^2(x)$: $\lim\limits_{x\to0} \dfrac{\sin^2(x)}{2\sin^2(x)} = \boxed{\dfrac{1}{2}}$.

Exercice 2 : Simplification et continuité (4,5 points)

4,5 pts

1) a) $\lim\limits_{x\to+\infty} f(x) = \lim\limits_{x\to+\infty} \dfrac{x}{x^2} = \boxed{0}$ et $\lim\limits_{x\to-\infty} f(x) = \lim\limits_{x\to-\infty} \dfrac{x}{x^2} = \boxed{0}$.

b) & c) $x^2 - x - 2 = (x-2)(x+1)$, donc pour $x \in \mathbb{R}\setminus\{-1, 2\}$: $f(x) = \dfrac{x-2}{(x-2)(x+1)} = \boxed{\dfrac{1}{x+1}}$.

D'où $\lim\limits_{x\to2} f(x) = \boxed{\dfrac{1}{3}}$, $\lim\limits_{x\to(-1)^-} \dfrac{1}{x+1} = \boxed{-\infty}$ et $\lim\limits_{x\to(-1)^+} \dfrac{1}{x+1} = \boxed{+\infty}$.

2) a) & b) $D_g = (\mathbb{R}\setminus\{-1, 2\}) \cup \{2\} = \boxed{\mathbb{R}\setminus\{-1\}}$.

• Pour $x > 2$, $g(x) = f(x) = \dfrac{1}{x+1} \Rightarrow \lim\limits_{x\to2^+} g(x) = \dfrac{1}{3} = g(2)$.

• Pour $x < 2$ $(x \neq -1)$, $g(x) = -f(x) = -\dfrac{1}{x+1} \Rightarrow \lim\limits_{x\to2^-} g(x) = -\dfrac{1}{3} \neq g(2)$.

Conclusion : g est continue à droite en 2 mais **n'est pas continue en 2**.

Exercice 3 : Calculs trigonométriques (4 points)

4 pts

• $A = \cos(\dfrac{\pi}{8}) + \cos(\dfrac{3\pi}{8}) + \cos(\pi - \dfrac{3\pi}{8}) + \cos(\pi - \dfrac{\pi}{8}) = \cos(\dfrac{\pi}{8}) + \cos(\dfrac{3\pi}{8}) - \cos(\dfrac{3\pi}{8}) - \cos(\dfrac{\pi}{8})$
$= \boxed{0}$.

• $B = \sin(\dfrac{2\pi}{5}) + \sin(\dfrac{4\pi}{5}) + \sin(2\pi - \dfrac{4\pi}{5}) + \sin(2\pi - \dfrac{2\pi}{5}) = \sin(\dfrac{2\pi}{5}) + \sin(\dfrac{4\pi}{5}) - \sin(\dfrac{4\pi}{5}) - \sin(\dfrac{2\pi}{5}) = \boxed{0}$.

• Comme $\dfrac{2\pi}{5} = \dfrac{\pi}{2} - \dfrac{\pi}{10}$ et $\dfrac{3\pi}{10} = \dfrac{\pi}{2} - \dfrac{\pi}{5}$: $C = [\cos^2(\dfrac{\pi}{10}) + \sin^2(\dfrac{\pi}{10})] + [\cos^2(\dfrac{\pi}{5}) + \sin^2(\dfrac{\pi}{5})] = 1 + 1 = \boxed{2}$.

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Exercice 4 : Équations et relations trigonométriques (4,5 points)

4,5 pts

1) $(\cos x - \sin x)^2 + (\cos x + \sin x)^2 = (\cos^2 x - 2\cos x \sin x + \sin^2 x) + (\cos^2 x + 2\cos x \sin x + \sin^2 x) = 2(\cos^2 x + \sin^2 x) = \boxed{2}$.

2) a) & b) On a $(\frac{\sqrt{2}}{2})^2 + (\cos x + \sin x)^2 = 2 \Leftrightarrow (\cos x + \sin x)^2 = 2 - \frac{1}{2} = \frac{3}{2}$. Comme $x \in [0 ; \frac{\pi}{2}[$, $\cos x + \sin x \geq 0$, donc $\cos x + \sin x = \sqrt{\frac{3}{2}} = \boxed{\frac{\sqrt{6}}{2}}$.

Par somme et différence : $2\cos x = \frac{\sqrt{6}+\sqrt{2}}{2} \Rightarrow \boxed{\cos x = \frac{\sqrt{6}+\sqrt{2}}{4}}$ et $2\sin x = \frac{\sqrt{6}-\sqrt{2}}{2} \Rightarrow \boxed{\sin x = \frac{\sqrt{6}-\sqrt{2}}{4}}$.

3) a) & b) $(\cos x - \sin x)^2 = \cos^2 x + \sin^2 x - 2\sin x \cos x = 1 - \sin 2x$. Ainsi $1 - \sin 2x = \frac{1}{2} \Leftrightarrow \boxed{\sin 2x = \frac{1}{2}}$.

Comme $2x \in [0 ; \pi]$ et $\cos x > \sin x$ (soit $x \in [0 ; \frac{\pi}{4}[$), on a $2x = \frac{\pi}{6}$, d'où $\boxed{x = \frac{\pi}{12}}$.

Exercice 5 : Identité trigonométrique (2 points)

2 pts

Pour tout $x \in \mathbb{R} \setminus \{\frac{k\pi}{2}, k \in \mathbb{Z}\}$: $\frac{\sin 3x}{\sin x} - \frac{\cos 3x}{\cos x} = \frac{\sin 3x \cos x - \cos 3x \sin x}{\sin x \cos x} = \frac{\sin(3x - x)}{\frac{1}{2}\sin 2x} = \frac{\sin 2x}{\frac{1}{2}\sin 2x} = \boxed{2}$.

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